REAL-VALUED FUNCTIONS OF TWO OR MORE VARIABLES

Two-variable function

In[1]:=
```
f[x_,y_]:= Sin[x^2 - y^2]
```

In[2]:=
```
f[0,Sqrt[Pi/4]]
```

Out[2]= $-\frac{1}{\sqrt{2}}$

In[3]:=
```
h = Sin[x^2-y^2]
```

Out[3]= $\text{Sin}[x^2 - y^2]$

Three-variable function

In[4]:=
```
g[x_,y_,z_]:=x^2 y^3 - 3 x z
```

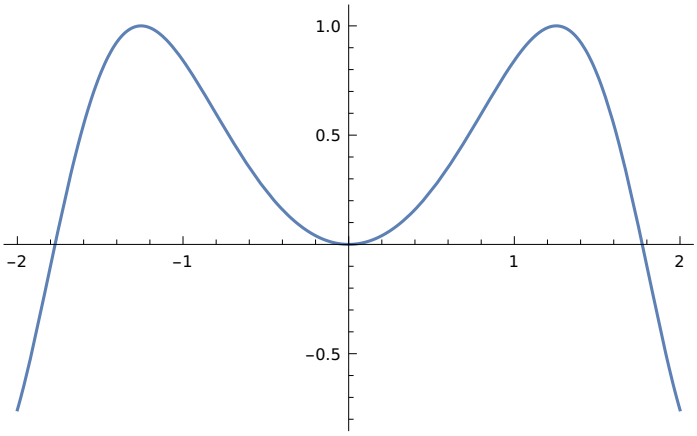
PLOTTING FUNCTIONS OF TWO AND MORE VARIABLES

- ***PLOT***

Putting y = 0

In[6]:=
```
Plot[f[x,0],{x,-2,2}]
```

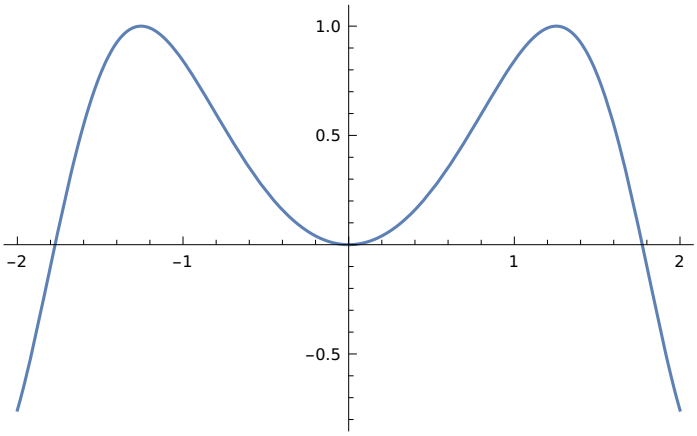
Out[6]=



Or we can use /. to put a value

In[7]:=
```
Plot[h/.y→0,{x,-2,2}]
```

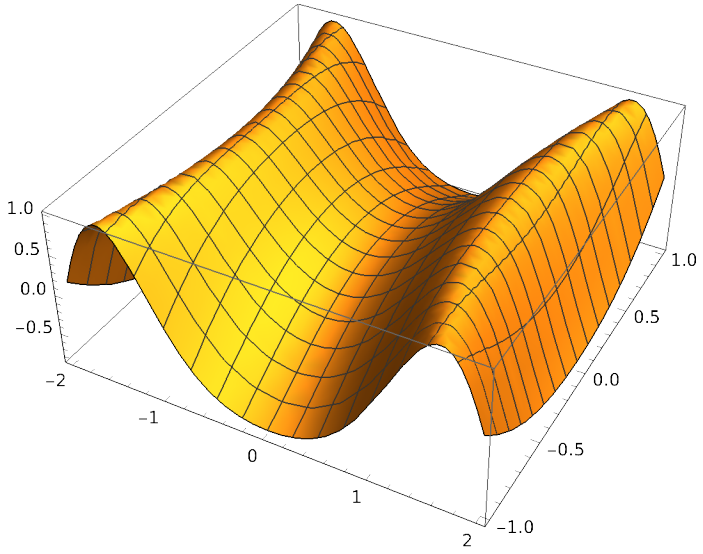
Out[7]=



- ***PLOT 3D***

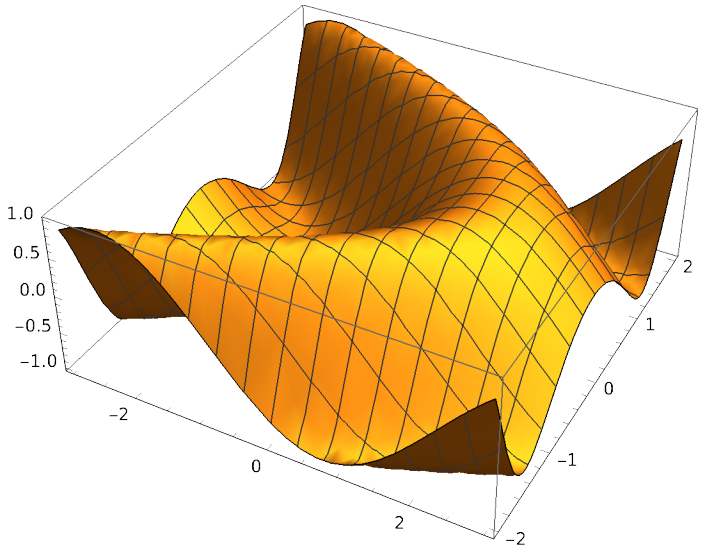
In[24]:=
```
Plot3D[h,{x,-2,2},{y,-1,1}]
```

Out[24]=



In[25]:=
```
Plot3D[Sin[x + y^2], {x, -3, 3}, {y, -2, 2}]
```

Out[25]=

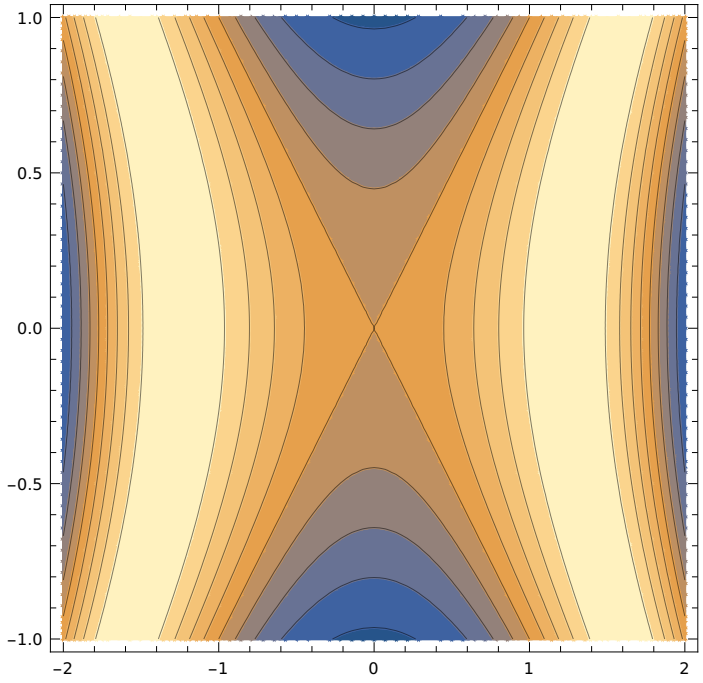


▪ *CONTOUR PLOT 3D*

For 2-variable function

In[8]:=
```
ContourPlot [f[x,y],{x,-2,2},{y,-1,1}]
```

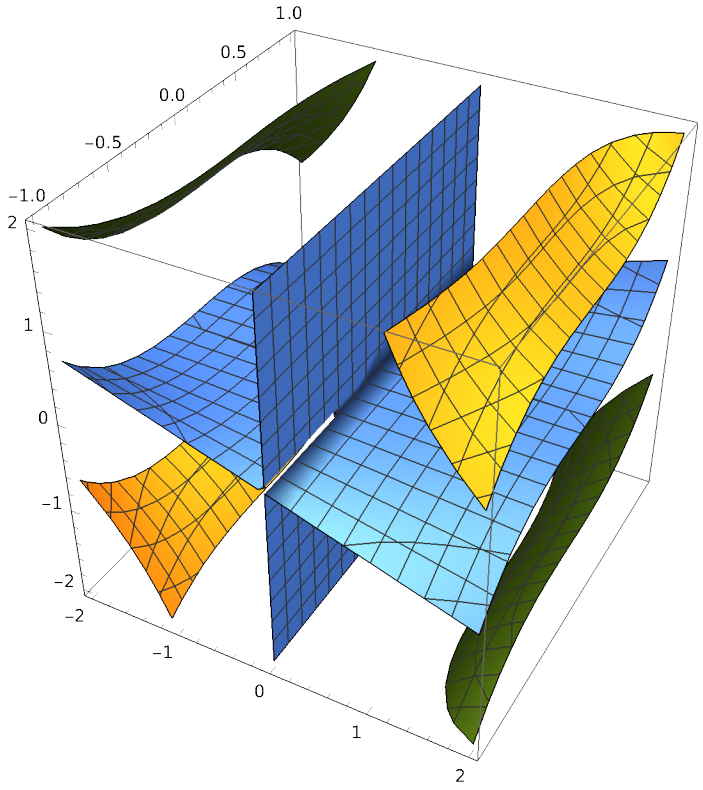
Out[8]=



For 3-variable function

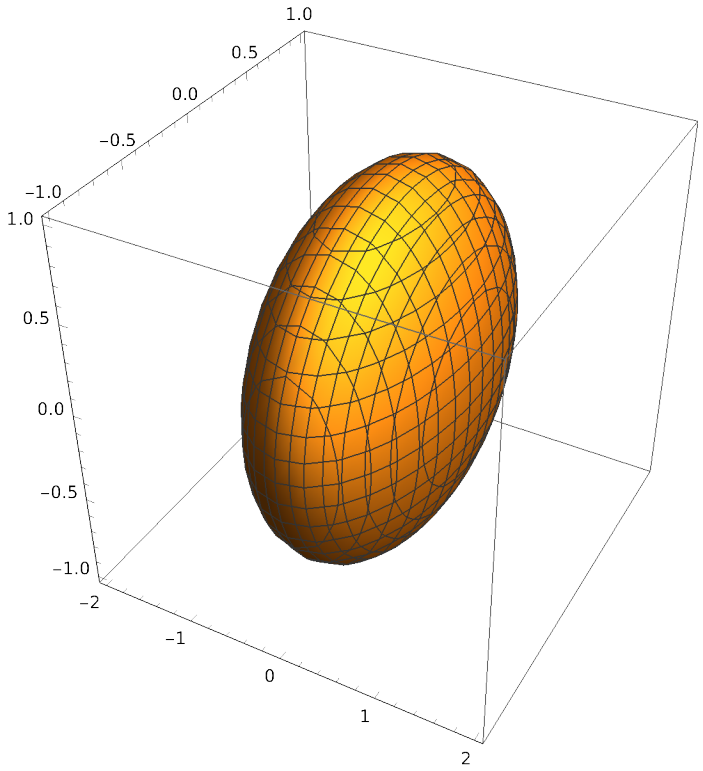
In[9]:=
```
ContourPlot3D [g[x,y,z],{x,-2,2},{y,-1,1},{z,-2,2}]
```

Out[9]=



In[10]:=
```
ContourPlot3D [x^2+y^2+z^2==1,{x,-2,2},{y,-1,1},{z,-1,1}]
```

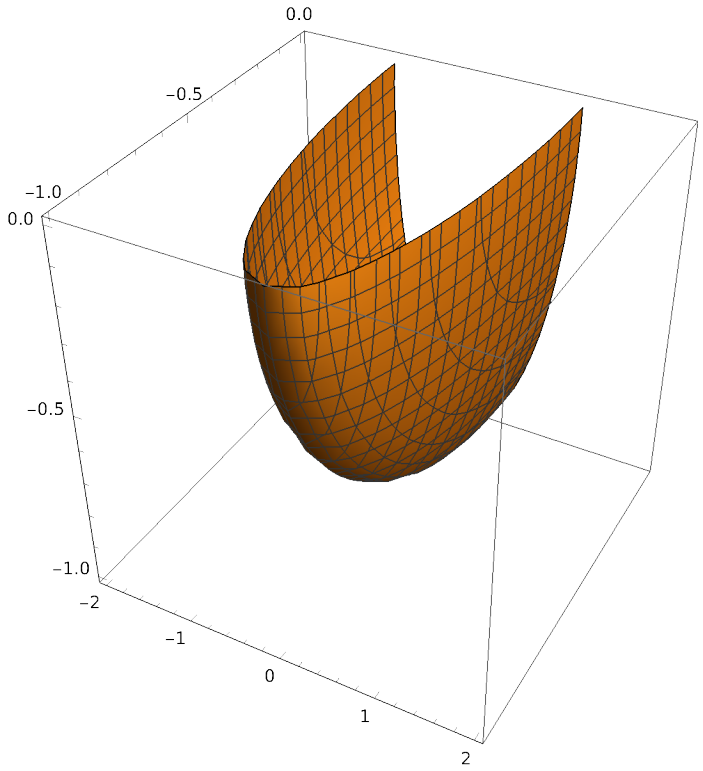
Out[10]=



Chanaging the range of one of the variables

In[11]:=
```
ContourPlot3D [x^2+y^2+z^2==1,{x,-2,2},{y,-1,0},{z,-1,0}]
```

Out[11]=



PARTIAL DIFFERENTIATION

Gives the partial derivative ∂h/∂x: differentiate h with respect to x

In[12]:=
```
D[h,x]
```

Out[12]= $2\,x\,\text{Cos}[x^2 - y^2]$

Gives the partial derivative ∂h/∂y: differentiate h with respect to y

In[13]:=
```
D[h,y]
```

Out[13]= $-2\,y\,\text{Cos}[x^2 - y^2]$

Gives the partial derivative (∂/∂y)(∂/∂x)h: differentiate h with respect to x first and then y

In[14]:=
```
D[h,x,y]
```

Out[14]= $4\,x\,y\,\text{Sin}[x^2 - y^2]$

Gives the partial derivative (∂/∂x)(∂/∂y)h: differentiate h with respect to y first and then x

In[15]:=
```
D[h,y,x]
```

Out[15]=
$$4\,x\,y\,\text{Sin}[x^2 - y^2]$$

Gives the multiple partial derivative (∂^4/∂^4 y)(∂^3/∂^3 x)h

In[16]:=
```
D[h,{x,3},{y,4}]
```

Out[16]=
$$-8\,x^3\,(-12\,\text{Cos}[x^2 - y^2] + 16\,y^4\,\text{Cos}[x^2 - y^2] - 48\,y^2\,\text{Sin}[x^2 - y^2]) -$$
$$12\,x\,(48\,y^2\,\text{Cos}[x^2 - y^2] - 12\,\text{Sin}[x^2 - y^2] + 16\,y^4\,\text{Sin}[x^2 - y^2])$$

OPTIMIZATION

▪ *MAXIMIZE*

In[17]:=
```
Maximize[-85+16 x-4 x^2-4 y-4 y^2+40 z-4 z^2, {x,y,z}]
```

Out[17]=
$$\left\{32, \left\{x \to 2, y \to -\frac{1}{2}, z \to 5\right\}\right\}$$

▪ *MINIMIZE*

Minimize the function subject to the constraints

In[18]:=
```
Minimize[{12 y^3+4 x^2-10 x y, -1≤x≤1&&-1≤y≤1},{x,y}]
```

Out[18]=
$$\{-18, \{x \to -1, y \to -1\}\}$$

▪ *NMINIMIZE*

Minimize the function subject to the constraints

In[20]:=
```
NMinimize[{12 y^3+4 x^2-10 x y, x^2+y^2≤1},{x,y}]
```

Out[20]=
$$\{-13.0925, \{x \to -0.215258, y \to -0.976557\}\}$$